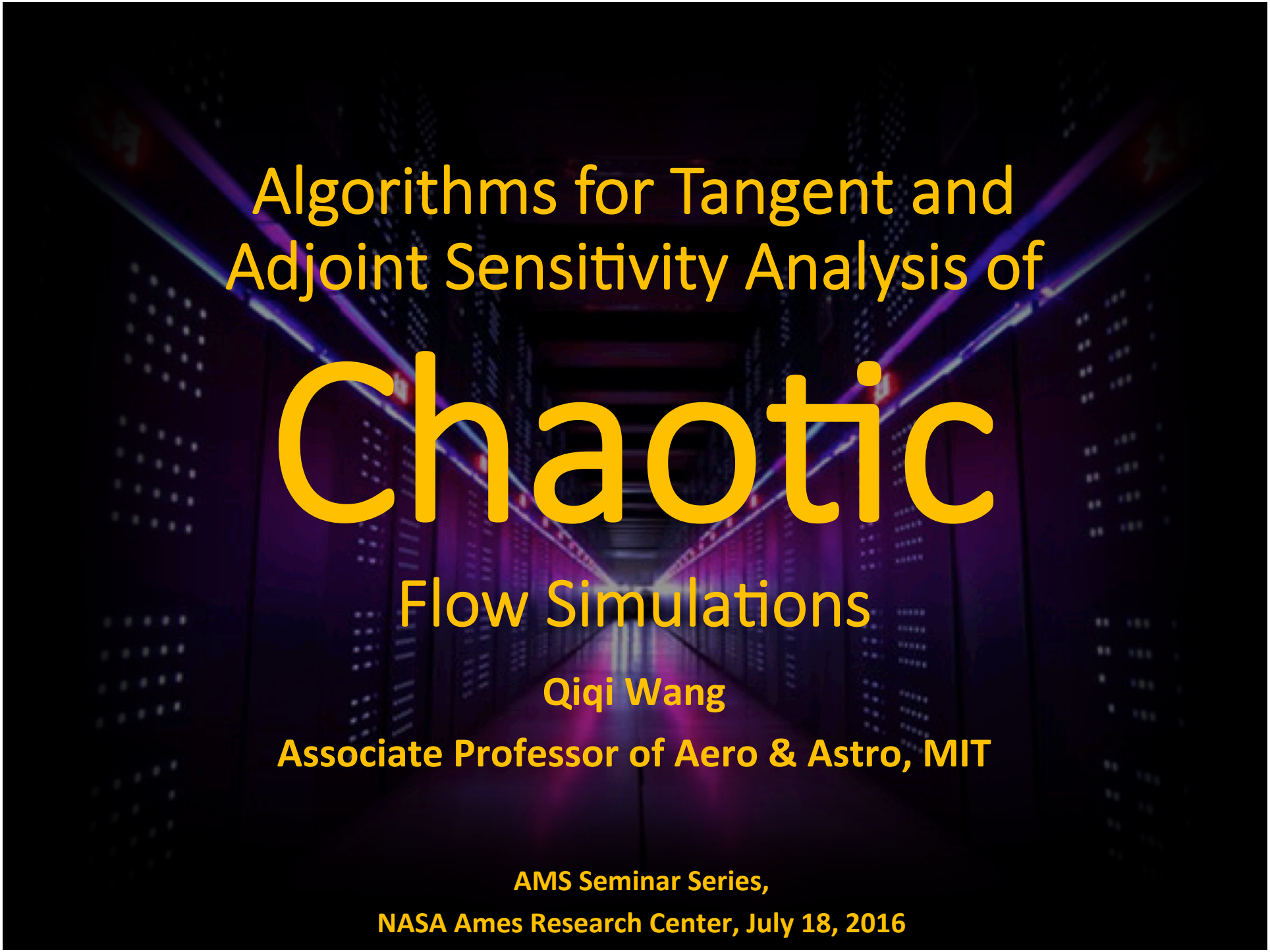

Algorithms for Tangent and Adjoint Sensitivity Analysis of Chaotic Flow Simulations

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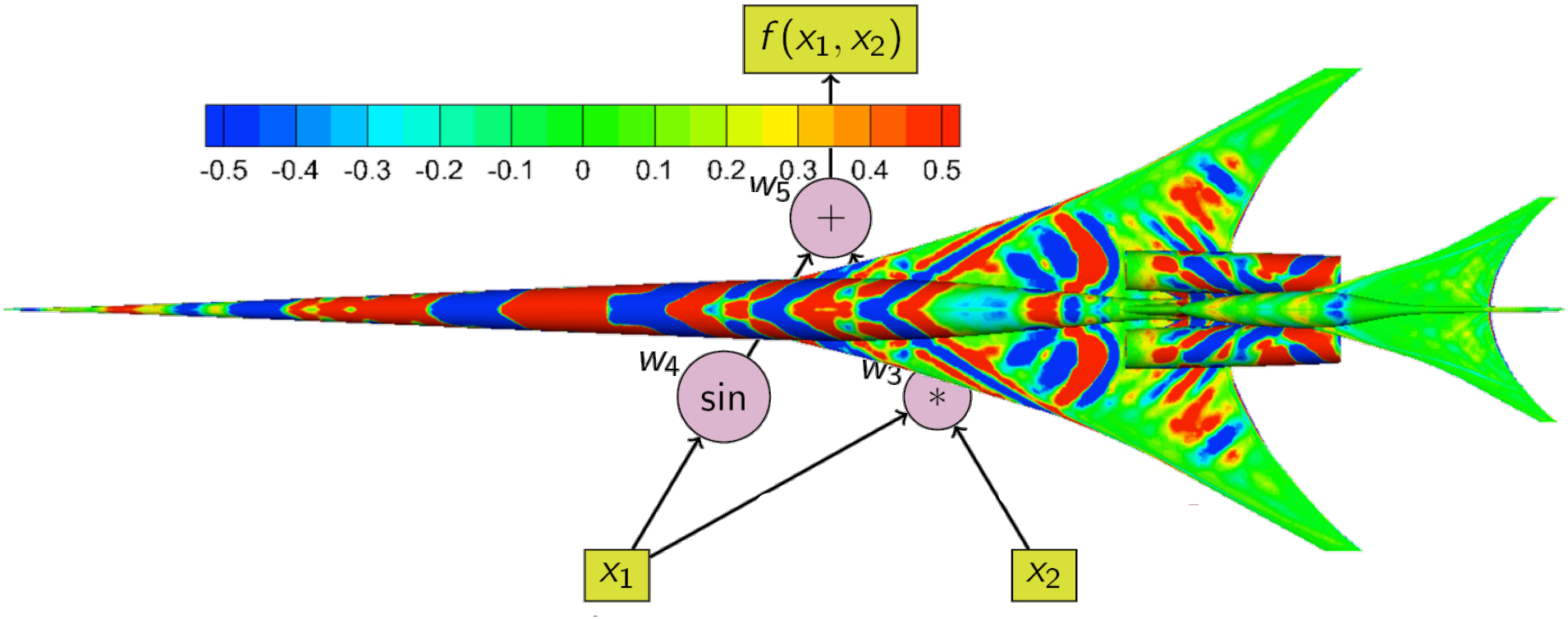
AMS Seminar Series,

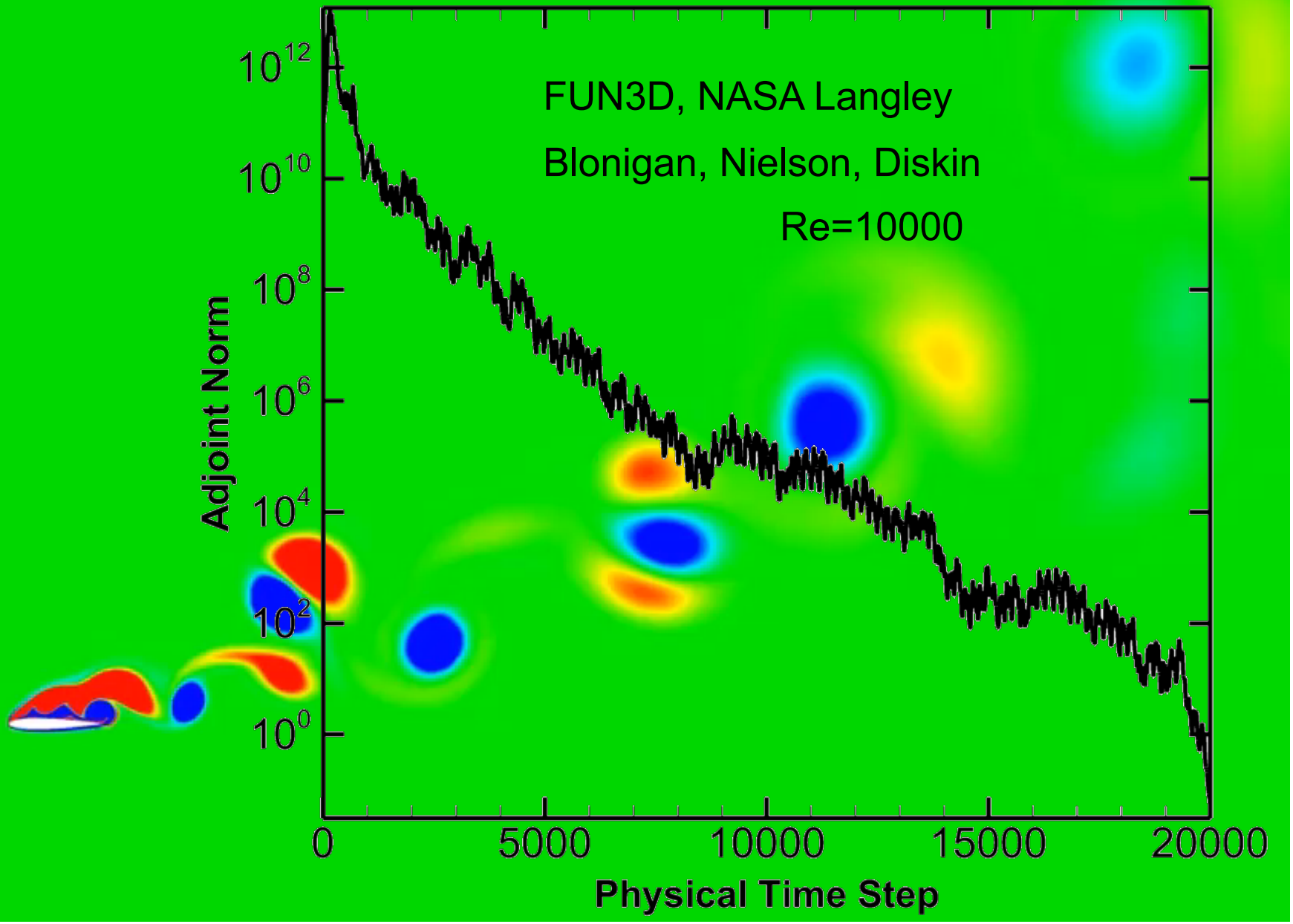
NASA Ames Research Center, July 18, 2016

Adjoin /əˈjoin/ verb: **differentiate the output of a simulation to its inputs** by tracing backwards through the calculations.



Rallabhandi, Nielsen, Diskin 2012 (FUN3D)
Derivative of the loudness of a supersonic airplane with respect to its shape

Diverging adjoint in **chaotic** separated flow

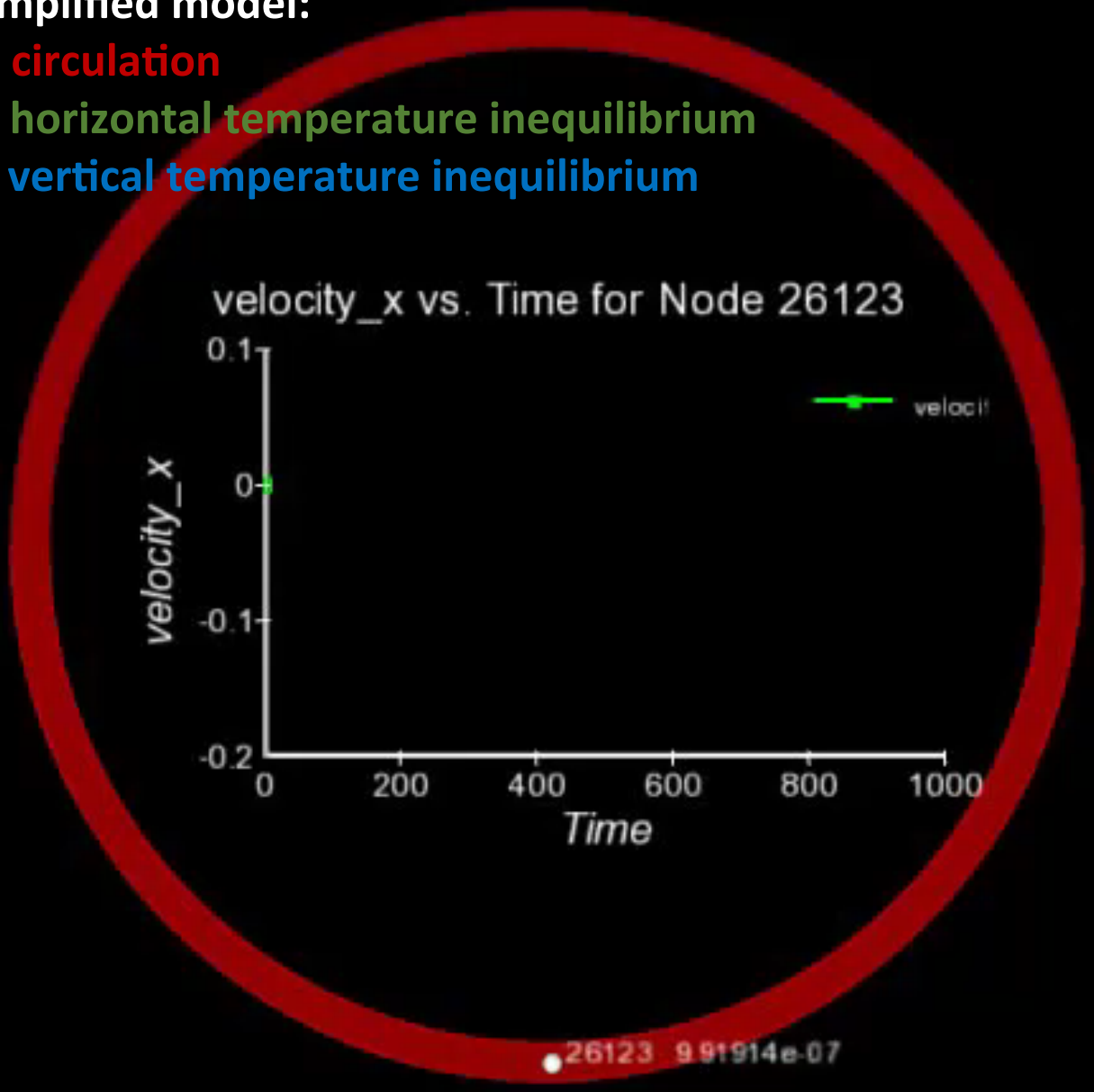


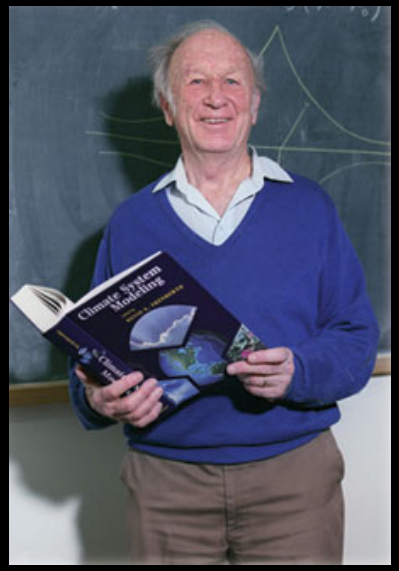
Simplified model:

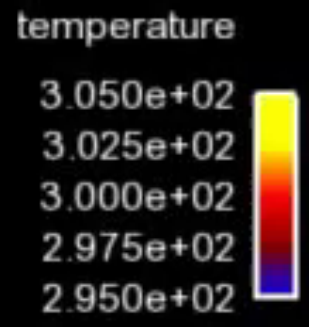
X: circulation

Y: horizontal temperature inequilibrium

Z: vertical temperature inequilibrium

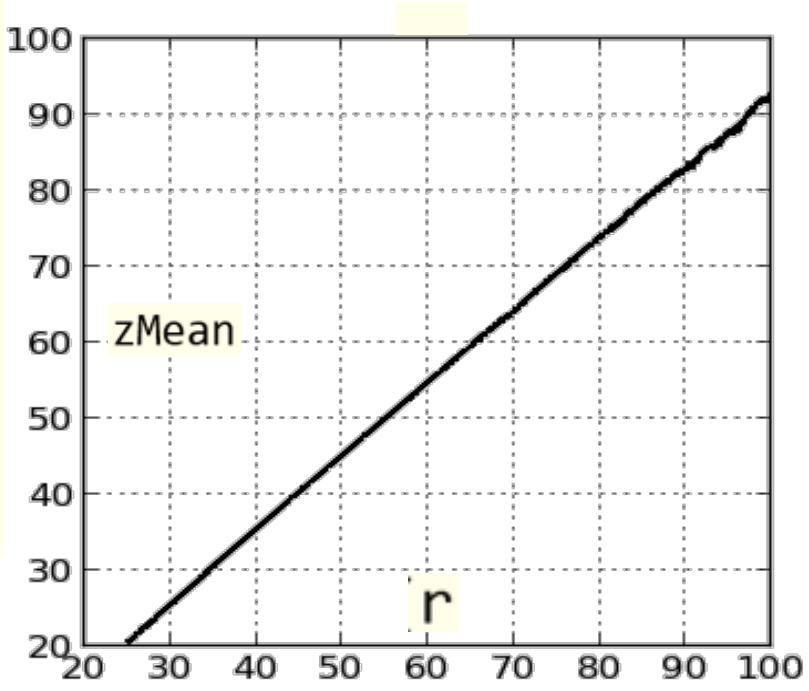






```
double lorenz(r)
{
    double b = 8./3., s = 10.;
    double T = 100., dt = 0.01;
    double x = 1., y = 1., z = 1., zMean = 0.;
    for (double t = 0; t < T; t += dt) {
        double dx = s * (y - x);
        double dy = x * (r - z) - y;
        double dz = x * y - b * z;
        x += dt * dx;
        y += dt * dy;
        z += dt * dz;
        zMean += dt * z / T;
    }
    return zMean;
}
```

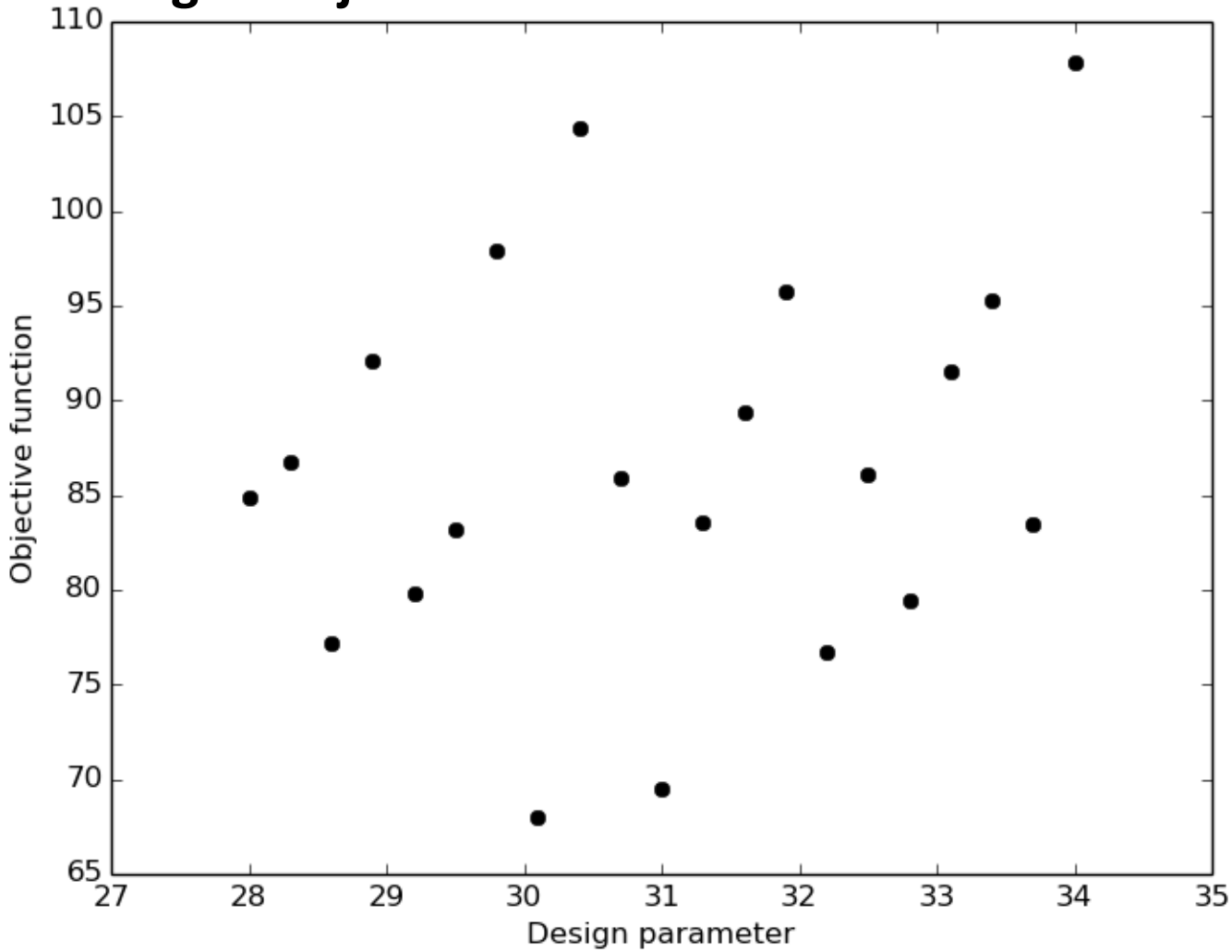
d(zMean)/dr=-**1.83E+23**



Lorenz system: solutions are sensitive small perturbations

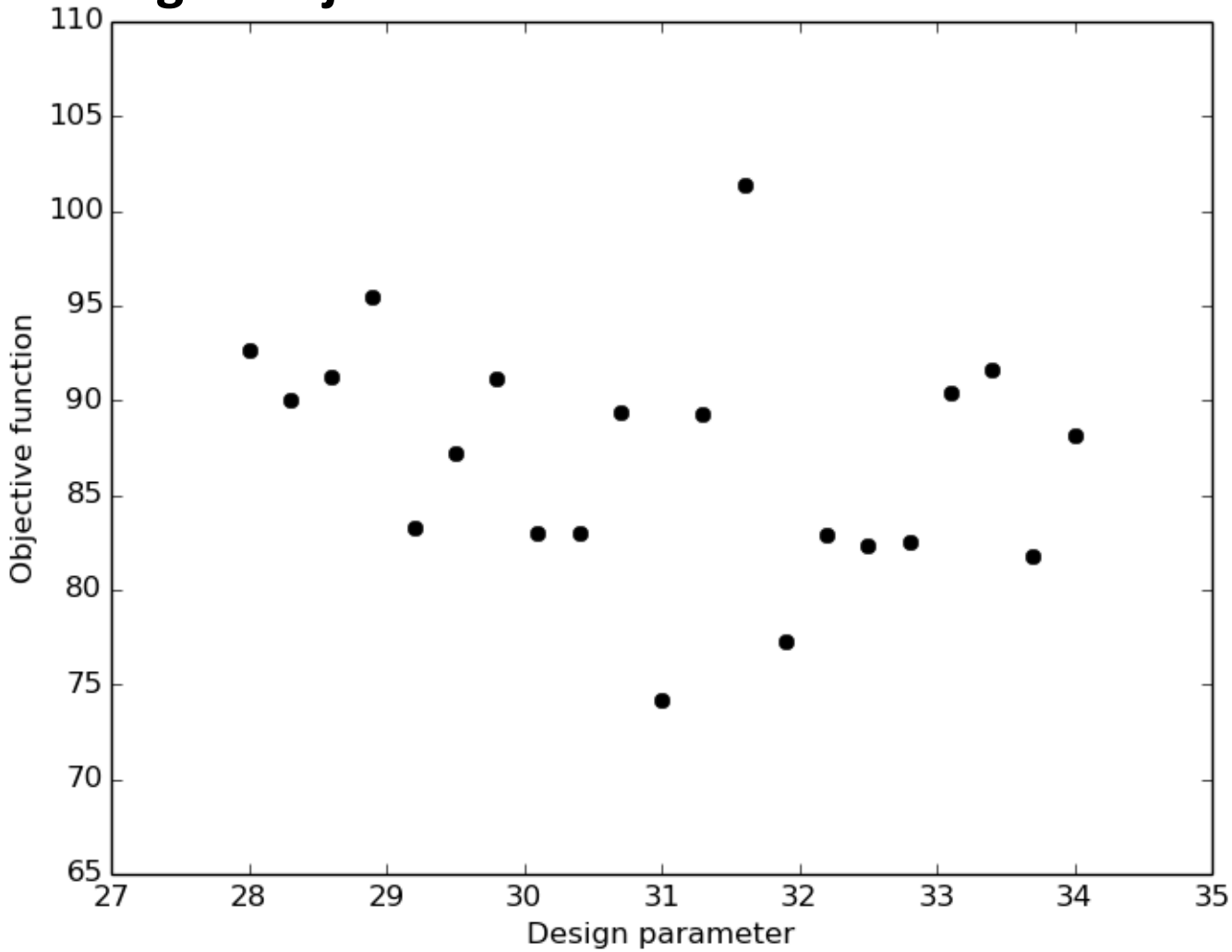
Smooth Response from Chaotic High-Fidelity Simulations

A time-averaged objective function in a chaotic simulation (T=5)



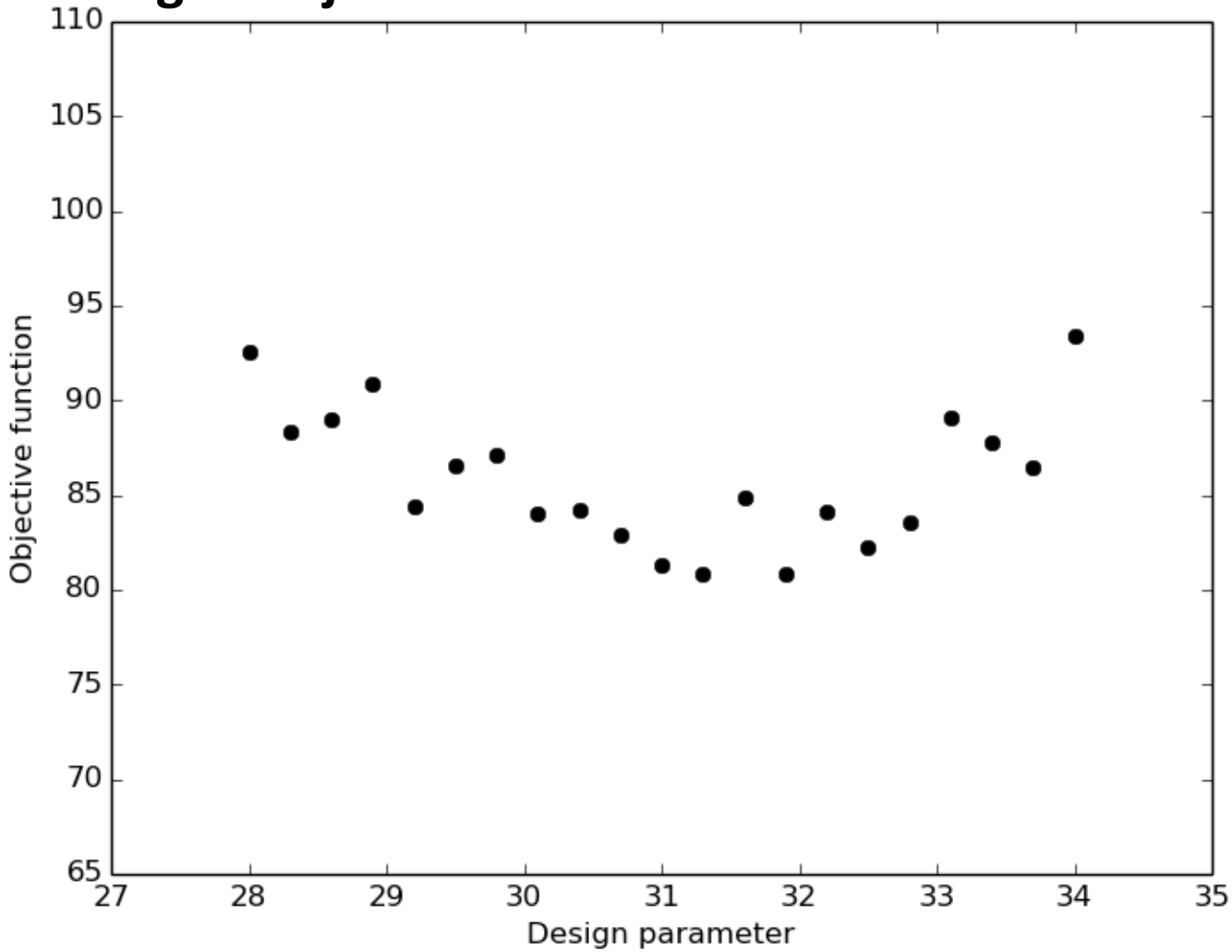
Smooth Response from Chaotic High-Fidelity Simulations

A time-averaged objective function in a chaotic simulation (T=15)



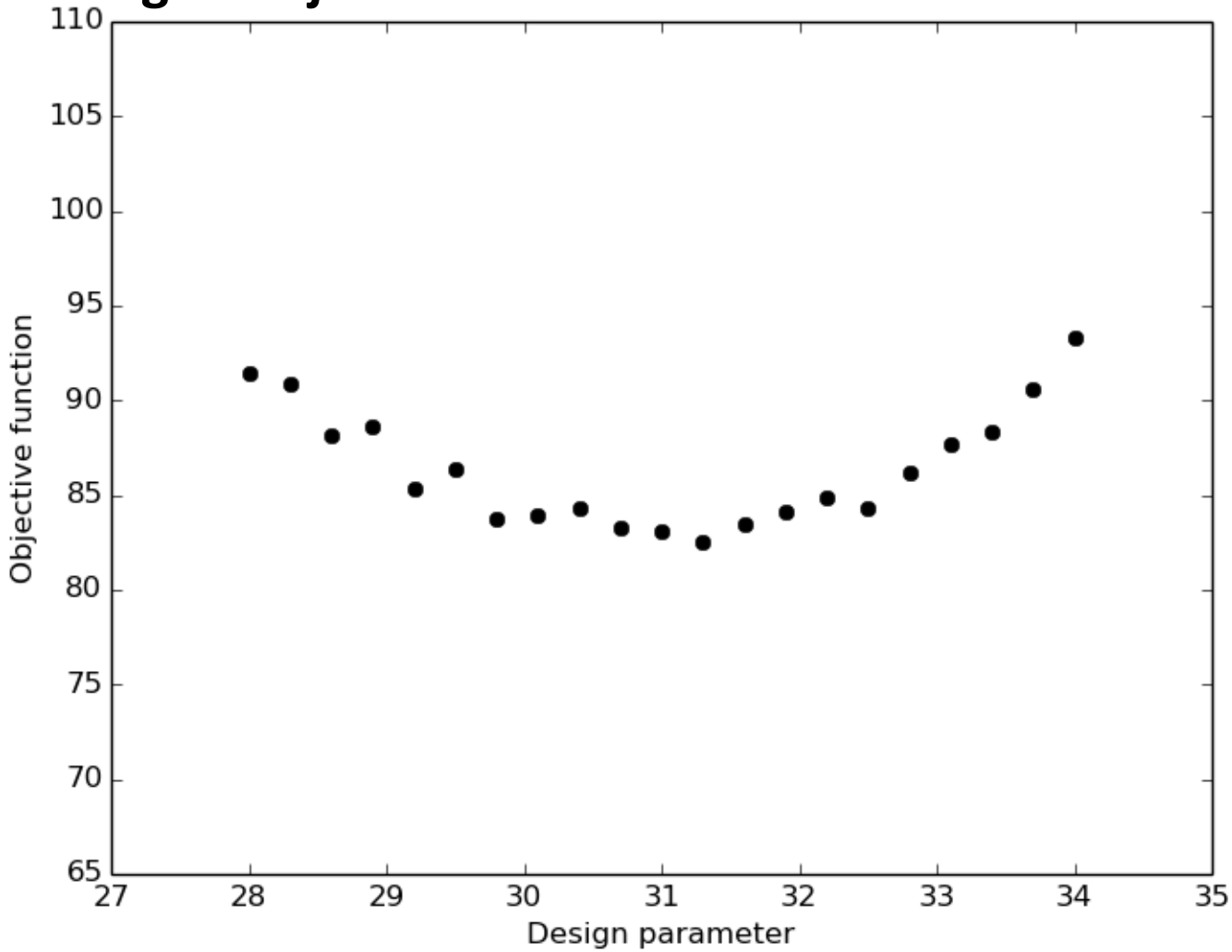
Smooth Response from Chaotic High-Fidelity Simulations

A time-averaged objective function in a chaotic simulation (T=50)



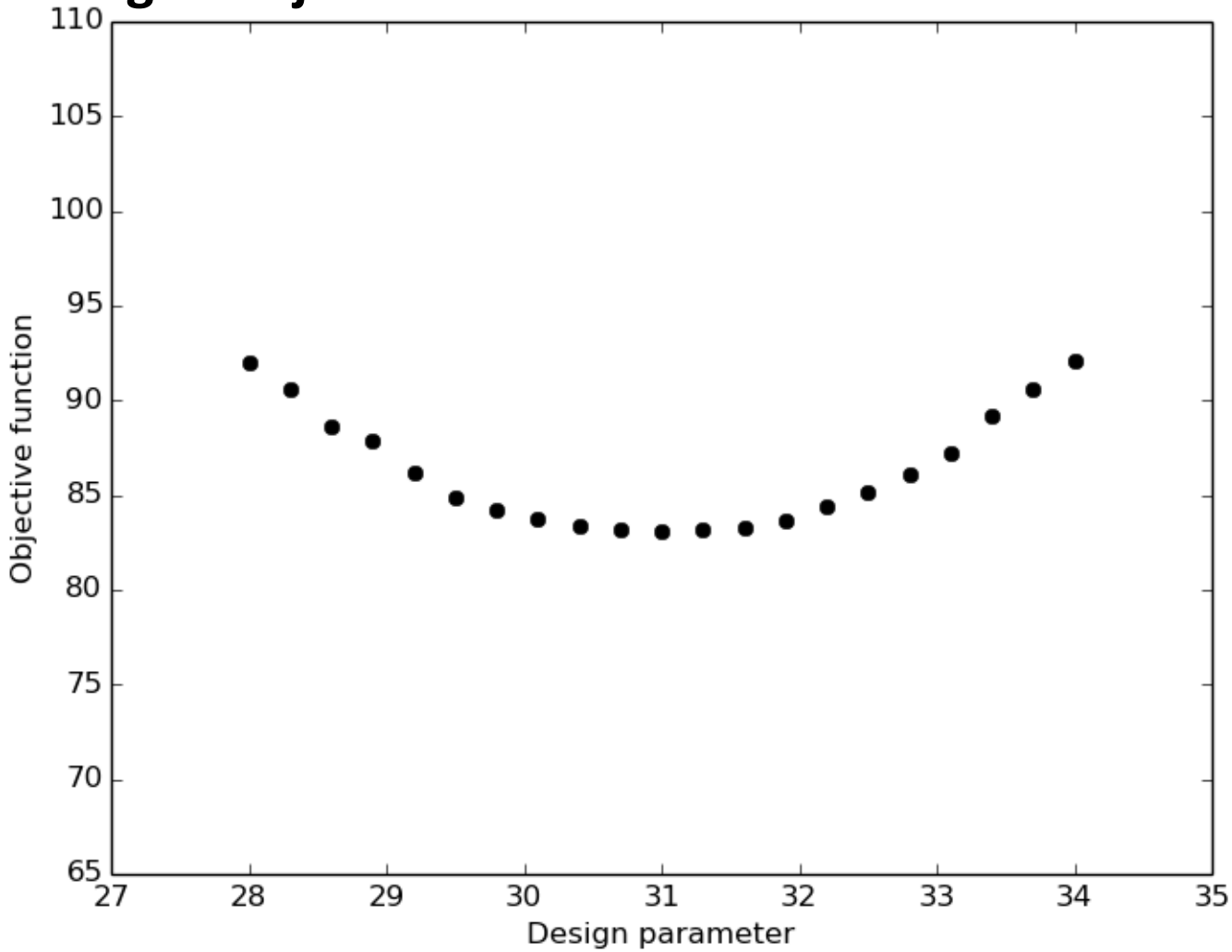
Smooth Response from Chaotic High-Fidelity Simulations

A time-averaged objective function in a chaotic simulation (T=500)



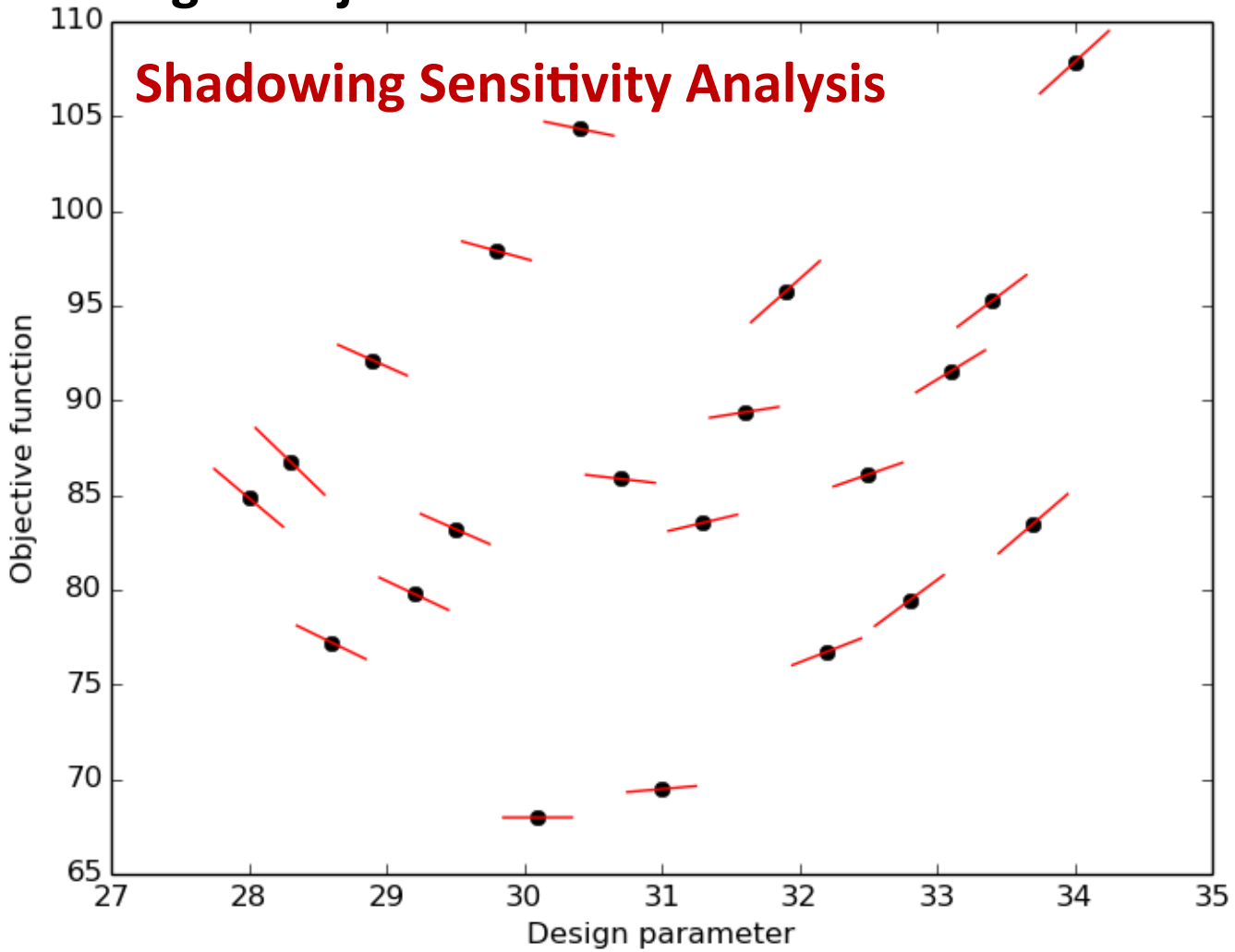
Smooth Response from Chaotic High-Fidelity Simulations

A time-averaged objective function in a chaotic simulation (T=5000)

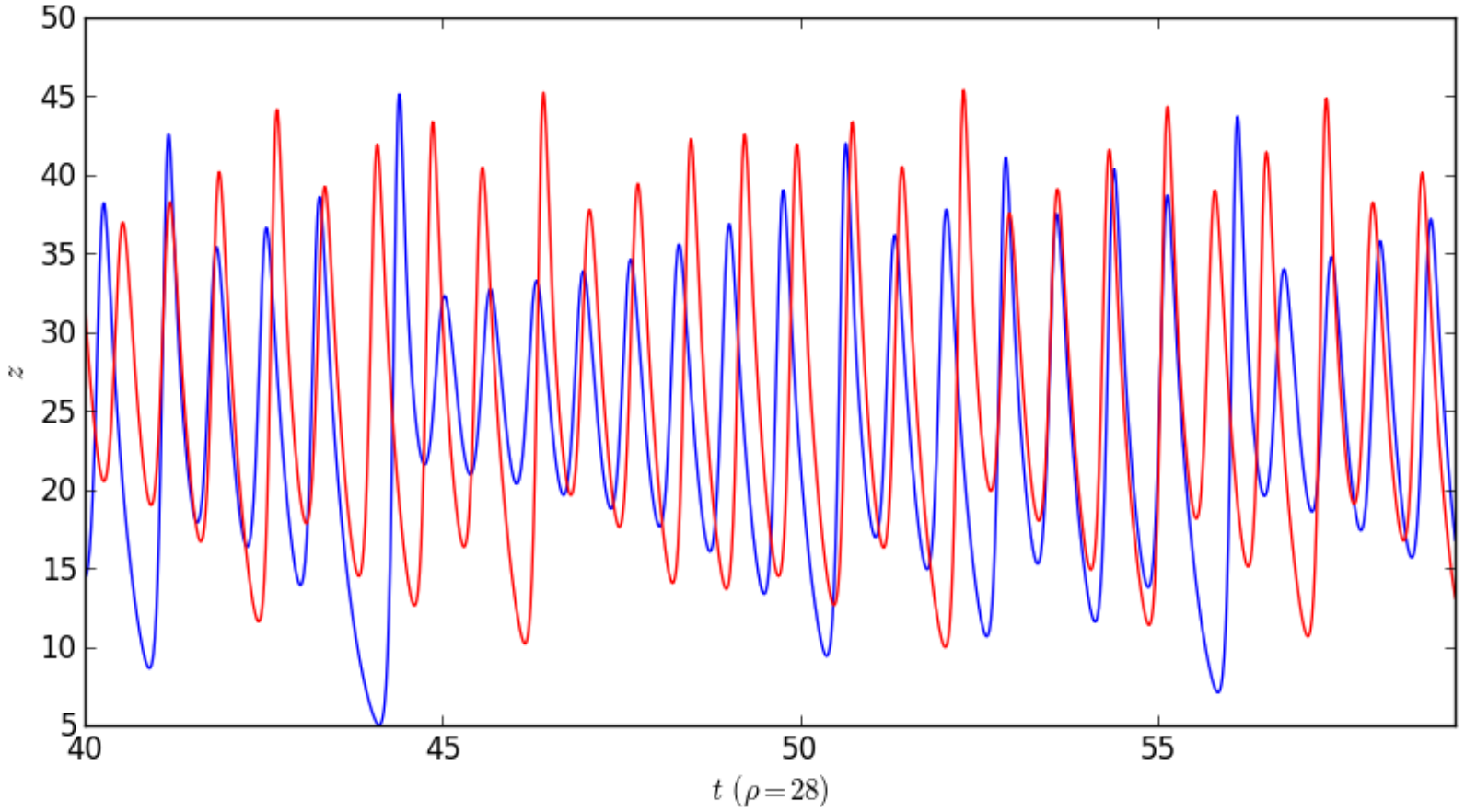


Smooth Response from Chaotic High-Fidelity Simulations

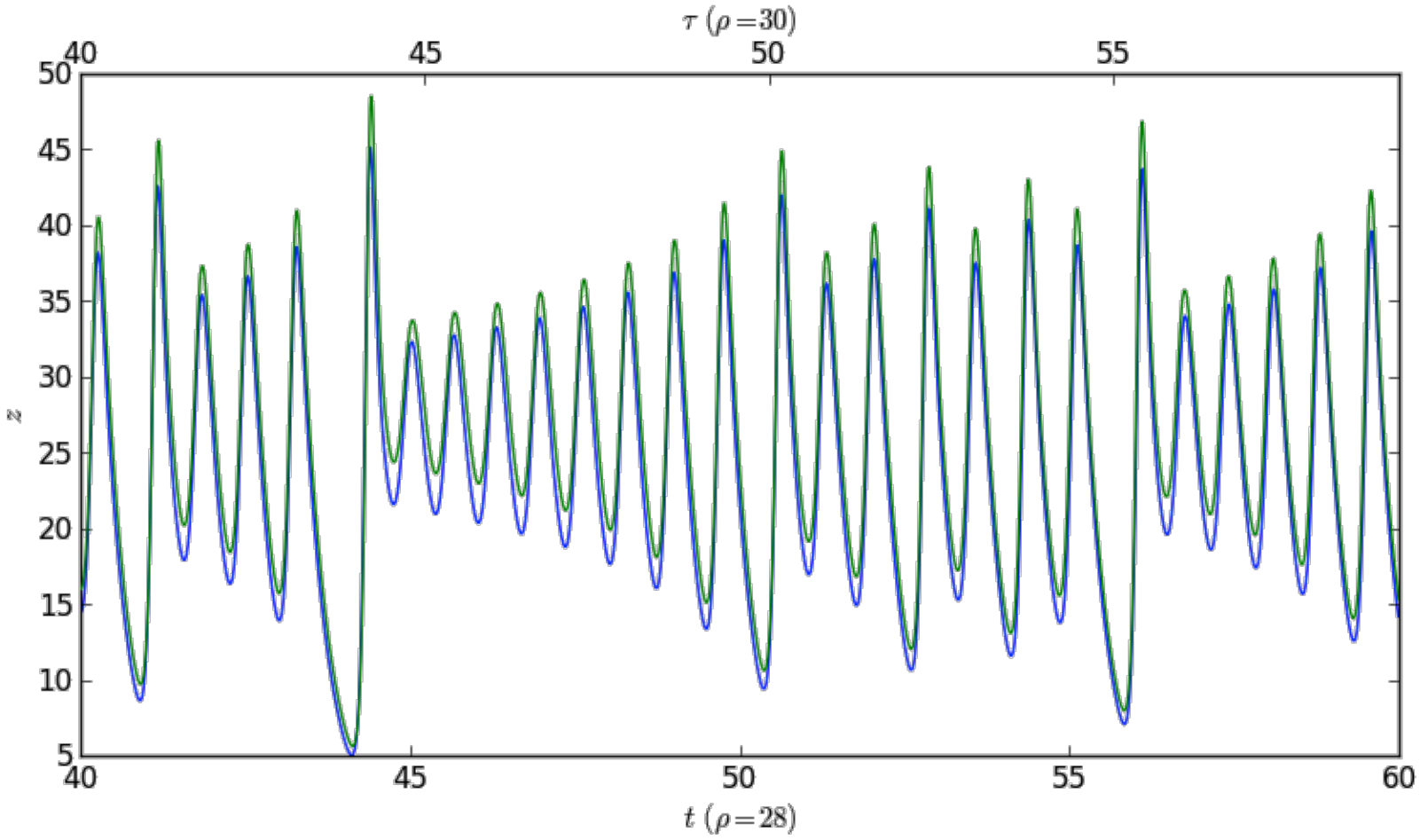
A time-averaged objective function in a chaotic simulation (T=5)



Trajectories with similar parameters diverge



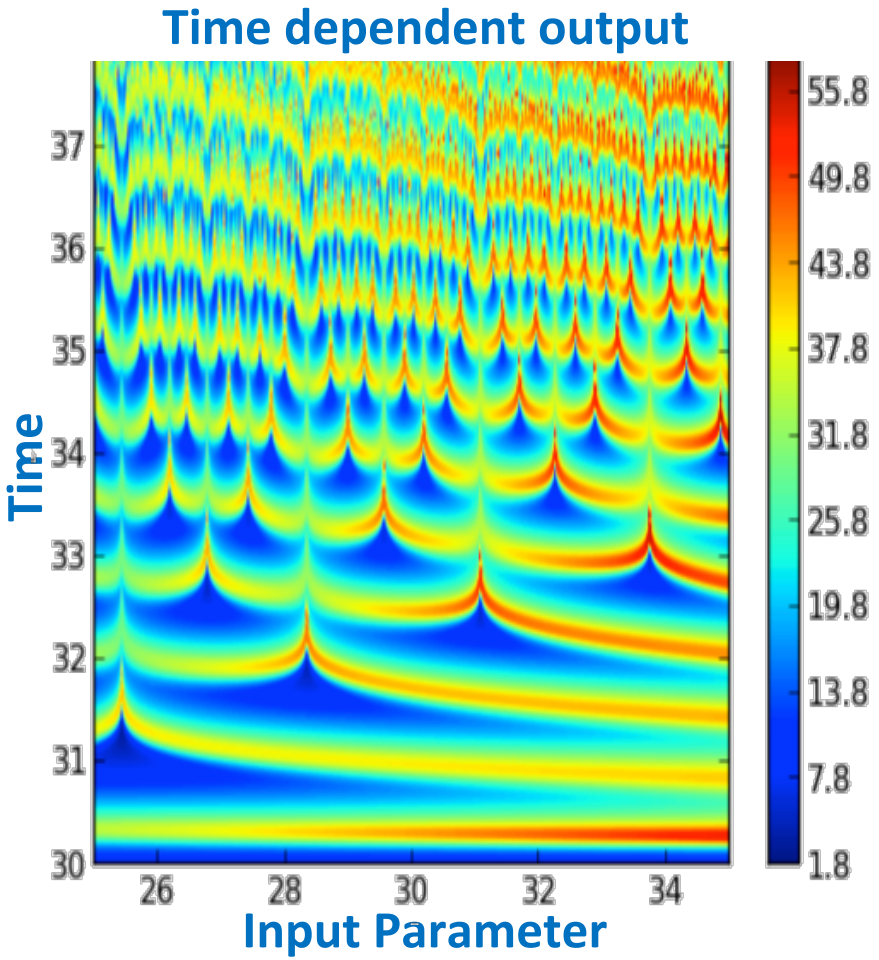
“Shadowing” Trajectories with similar parameters and carefully adjusted initial conditions do not diverge

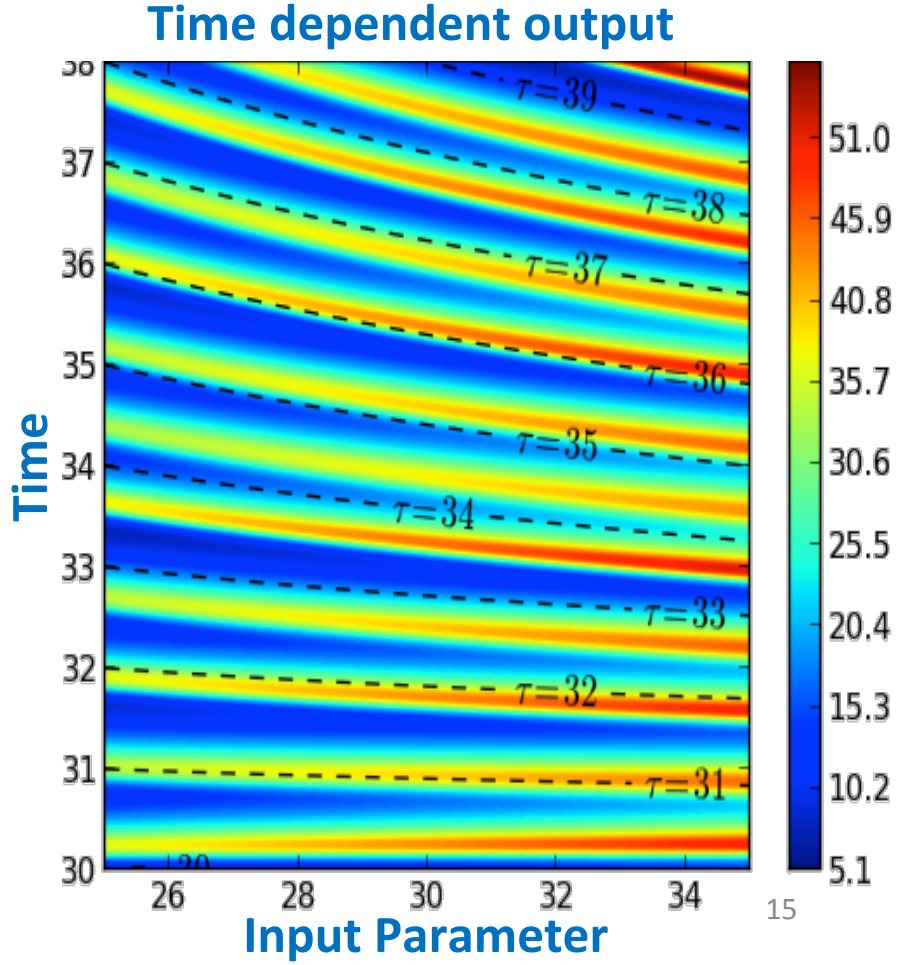


Least Squares Shadowing:
Replace the initial condition with least squares

$$\min \frac{1}{2}\int_{T_0}^{T_1} \|u(\tau(t)) - u_r(t)\|^2 dt \qquad \text{s.t.} \quad \frac{du}{d\tau} = f(u; s+\delta s)$$

Reference solution

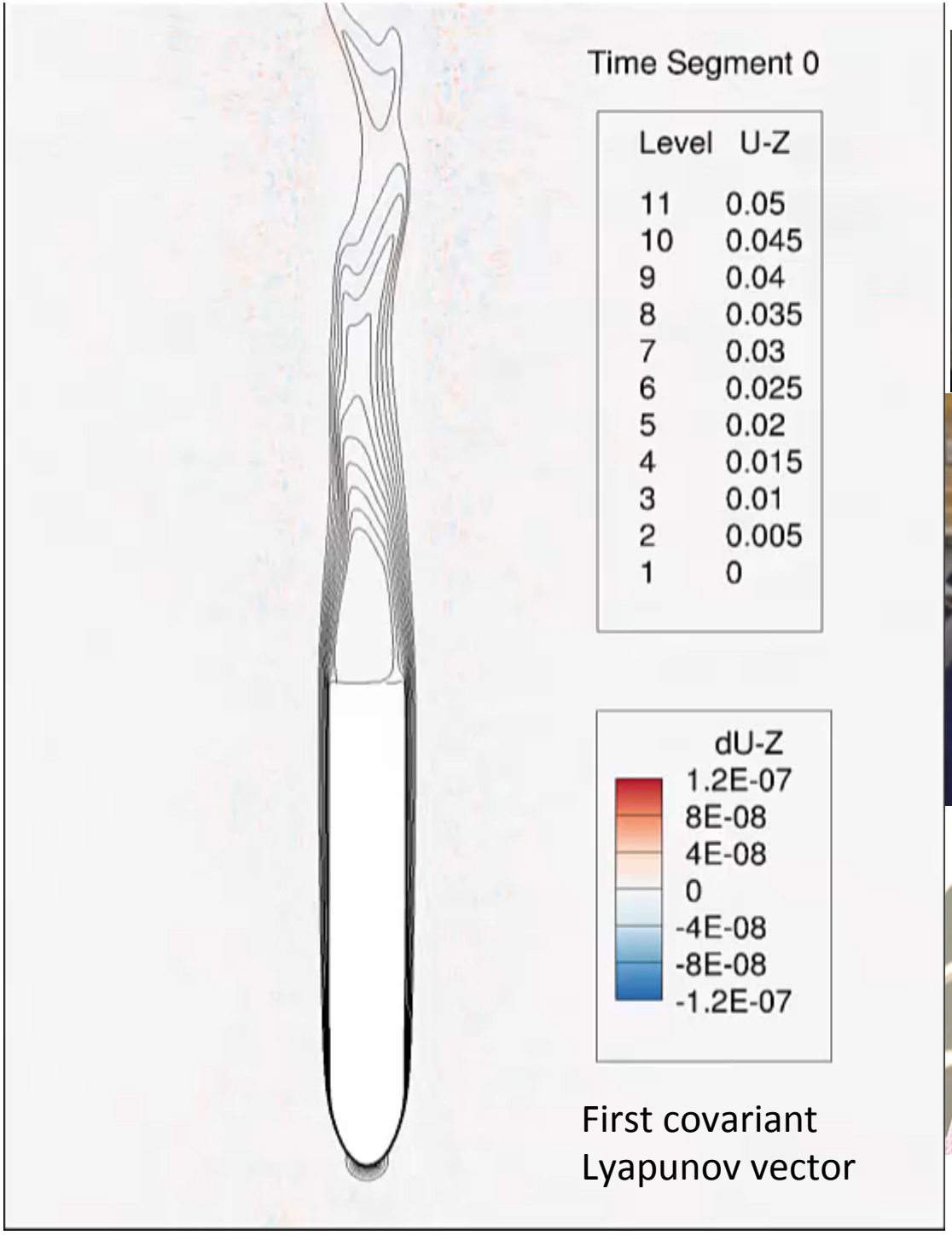




Oseledets theorem

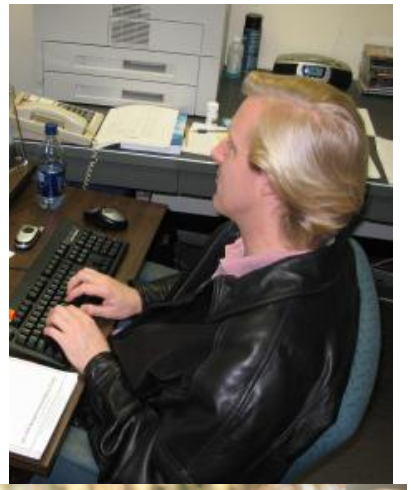
Benettin’s algorithm

Ginelli’s algorithm

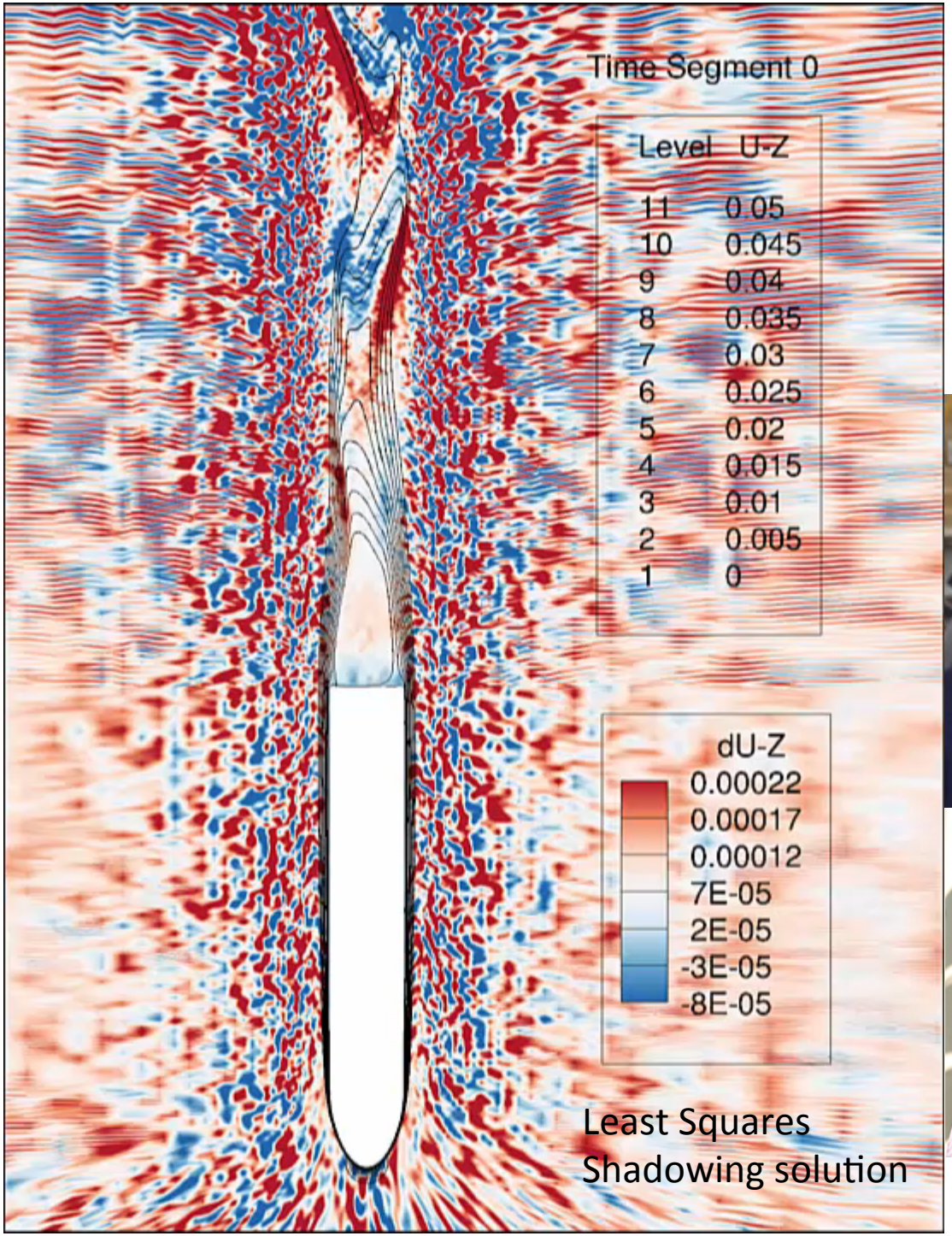


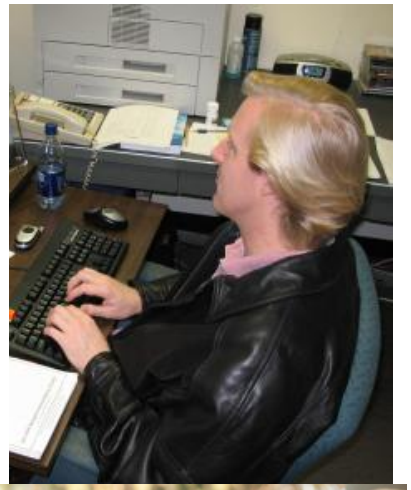
First covariant Lyapunov vector



Non-intrusive Least Squares Shadowing


Time Segment 0
Level U-Z
11 0.05
10 0.045
9 0.04
8 0.035
7 0.03
6 0.025
5 0.02
4 0.015
3 0.01
2 0.005
1 0
dU-Z
0.00022
0.00017
0.00012
7E-05
2E-05
-3E-05
-8E-05
Least Squares
Shadowing solution



Adjoint Least Squares Shadowing

Sensitivity Analysis of Chaotic Simulations

High-fidelity design in many important applications requires sensitivity analysis of chaotic simulations

Adjoint-based sensitivity analysis works, mathematically and practically, on chaotic simulations

Chaotic adjoint currently being implemented and tested on a state-of-the-art CFD solver (FUN3D)